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NONABELIAN BERRY PHASES IN BARYONS

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ABSTRACT

We show how generic nonabelian gauge fields can be induced in baryons when a hierarchy of fast degrees of freedom is integrated out. We identify them with nonabelian Berry potentials and discuss their role in transmuting quantum numbers in bag and soliton models of baryons. The resulting baryonic spectra both for light and heavy quark systems are generic and resemble closely the excitation spectrum of diatomic molecules. The symmetry restoration in the system, *i.e.* the electronic rotational invariance in diatomic molecules, the heavy-quark symmetry in heavy baryons etc. is interpreted in terms of the vanishing of nonabelian Berry potentials that otherwise govern the hyperfine splitting.

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1 Introduction

Whenever a quantum system with a hierarchy of length scales is truncated, induced gauge potentials are naturally generated reflecting on the degrees of freedom that are integrated out. A natural setting for discussing these issues has been discovered by Berry [1] in simple quantum systems responding to slowly varying external parameters. He showed that abelian magnetic monopoles naturally arise in the space of the slow variables due to degeneracy points. This concept has been generalized by Wilczek and Zee [2] to the nonabelian case. They have shown that if a set of degenerate energy levels depends on adiabatically varying external parameters, nonabelian gauge potentials are induced, affecting dynamics in a nontrivial way. Such gauge potentials have attracted a lot of attention in recent years because of their fundamental character and growing importance in quantum systems.

The concept of induced gauge fields has led to important understanding of subtle effects ranging from condensed matter to elementary particle physics [3]. Whenever the underlying dynamics can be separated into "slow" and "fast" degrees of freedom, induced gauge fields are generally expected. They are generic and embody the essence of the geometrical symmetries in a given problem. One may therefore ask whether similar gauge structures are encountered in models of strong interaction physics, and to what extent they bear on our understanding of hadronic physics.

In a series of recent papers [4], we have shown that nonabelian Berry structures can and do appear naturally in topological chiral bags [5] that model spontaneously broken chiral symmetry and confinement of QCD [1]. The distinction between "fast" and "slow" degrees of freedom is somehow blurred in the topological bag model. However, if we were to assume that the external pion field can be decomposed into a classical and quantum part, then a semiclassical delineation is possible in which the "slow" degrees of freedom refer to the large component of the fields and "fast" degrees of freedom refer to the small component of the field. In the semiclassical limit the bag is composed of a classical pion field that wraps valence quarks (bound fermions) and polarizes the Dirac sea. In this limit neither isospin nor angular momentum are good quantum numbers. Corrections to this limit are down by $\hbar$ and correspond to quantum external pions and (multi) quark-antiquark excitations each of which have good isospin and angular momentum assignment.

In the semiclassical quantization, the classical bag is adiabatically rotated generating states of good spin and isospin. Even when ignoring the quantum corrections the adiabatic

[1] While the topological bag model has confinement and the bag boundary condition plays an essential role in [4], we suspect that confinement is not really necessary for generating gauge structure and that it is only the symmetry that is relevant.

quantization does not reduce simply to the quantization of a spinning particle in isospin and spin space as for ordinary classical fields. Indeed, the degeneracy of the Dirac spectrum, following the symmetries of the classical field, implies that under any rotation (even if infinitesimally small) the quarks in the valence orbitals and the Dirac sea mix inside degenerate bands and between crossing levels. Adiabatic quark mixing is at the origin of the Berry phases in models of strong interactions. Below we will analyze their occurrence and physical relevance in the context of the topological bag model.

This paper is organized as follows. In section 2, we discuss the general setting for Berry phases in Born-Oppenheimer approximation and we demonstrate explicitly the Berry phase for the case of diatomic molecule. We show how the integration of the electronic (fast) degrees of freedom leads to the extra gauge potential-like term in the effective Hamiltonian for the nuclear (slow) degrees of freedom. This term causes splittings of the energy levels and changes quantum numbers of the system.

In section 3, we show how these concepts extend to the topological bag model. Subsection 3.1 includes the toy model displaying all the features of the topological bag model. Subsection 3.2 is devoted to the bag model itself. The role of the fast variables is played by the sea quarks inside the bag, whereas the adiabatic rotation of the solitonic cloud surrounding the bag constitutes the slow motion. We detail the explicit Hamiltonian construction for excited baryons in the light-quark sector and discuss in subsection 3.3 model-independent mass relations. In section 4, we show how the analogous construction of the Berry phases can be made in the context of soliton-heavy meson system. Integrating out heavy meson degrees of freedom we end up with the usual Skyrme-like rotor term, however, submitted to the influence of a non-trivial magnetic field (non-abelian Berry phase). This monopole-like field is responsible for the spin-isospin transmutation of the quantum numbers and for the structure of the hyperfine splittings. The independent mass relations are identical to the ones obtained in the framework of the Callan-Klebanov model of hyperon skyrmions [6].

In section 5, we discuss what we believe happens to the skyrmion structure associated with induced gauge fields when the heavy meson becomes infinitely heavy at which the recently discovered heavy-quark symmetry [7] is operative. Our major conclusions and prospects are relegated to section 6. In Appendix A, we give a heuristic reason based on an argument by Aharanov et al [8] why nonabelian Berry potentials cannot vanish in light-quark systems in contrast to diatomic molecules and heavy-quark baryons. In Appendix B, an argument is provided as to how heavy mesons decouple from the Wess-Zumino term responsible for the binding of heavy mesons to the soliton.

2 Berry Phases and the Born-Oppenheimer Approximation

To define the general setting for Berry phases and help understand their emergence in the context of the models of elementary particles, we will first present, following [9, 10] the pedagogical example of the induced gauge fields (Berry phases) in the Born-Oppenheimer approximation [11]. This approximation is usually described as a separation of slow (nuclear) and fast (electronic) degrees of freedom. This separation is motivated by the fact that the rotation of the nuclei does not cause the transitions between the electronic levels. In other words, the splittings between the fast variables are much bigger than the splittings between the slow ones. We will demonstrate how the integration of the fast degrees of freedom leads to the induced vector potential of the Dirac monopole affecting the dynamics of the slow motion. To make our analysis more quantitative, we define the generic Hamiltonian. Generically, the Hamiltonian is given by :

$$H = \frac{\vec{P}^2}{2M} + \frac{\vec{p}^2}{2m} + V(\vec{R}, \vec{r}) \tag{1}$$

where we have reserved the capitals for the slow variables and lower-case letters for the fast variables. We expect the electronic levels to be stationary under the adiabatic (slow) rotation of the nuclei. We split therefore the Hamiltonian into the fast and slow part,

$$\begin{aligned} H &= \frac{\vec{P}^2}{2M} + h \\ h(\vec{R}) &= \frac{\vec{p}^2}{2m} + V(\vec{r}, \vec{R}) \end{aligned} \tag{2}$$

where the fast Hamiltonian h depends *parametrically* on the slow variable $\vec{R}$. The snapshot Hamiltonian (for fixed $\vec{R}$) leads to the Schrödinger equation:

$$h\phi_n(\vec{r}, \vec{R}) = \epsilon_n(\vec{R})\phi_n(\vec{r}, \vec{R}) \, . \tag{3}$$

The wave function for the whole system is

$$\Psi(\vec{r}, \vec{R}) = \sum_n \Phi_n(\vec{R})\phi_n(\vec{r}, \vec{R}) \, . \tag{4}$$

Substituting the wave function into the full Hamiltonian and using the equation for the fast variables we get

$$\sum_n \left[\frac{\vec{P}^2}{2M} + \epsilon_n(\vec{R}) \right] \Phi_n(\vec{R})\phi_n(\vec{r}, \vec{R}) = E \sum_n \Phi_n(\vec{R})\phi_n(\vec{r}, \vec{R}) \tag{5}$$

where E is the energy of the whole system. Note that the operator of the kinetic energy of the slow variables acts on *both* slow and fast part of the wavefunction. We can now integrate over the fast degrees of freedom. A simple algebra leads to the following effective Schrödinger equation

$$\sum_m H^{eff}_{nm}\Phi_m = E\Phi_n \tag{6}$$

where the explicit form of the matrix-valued Hamiltonian (with respect to the fast eigenvectors) is

$$H^{eff}_{mn} = \frac{1}{2M}\sum_k \vec{\Pi}_{nk}\vec{\Pi}_{km} + \epsilon_n\delta_{nm} \tag{7}$$

where

$$\vec{\Pi}_{nm} = \delta_{nm}\vec{P} - i < \phi_n(\vec{r},\vec{R})|\vec{\nabla}_r|\phi_m(\vec{r},\vec{R}) > \equiv \delta_{mn}\vec{P} - \vec{A}_{nm}\ . \tag{8}$$

The above equation is exact. We see that the fast variables act like a gauge field. The vector part couples minimally to the momenta, and the fast eigenvalue acts like the scalar potential.

In the adiabatic approximation one may neglect the off-diagonal transition terms in the induced gauge potentials, which leads to the simpler Hamiltonian

$$H^{eff}_n = \frac{1}{2M}(\vec{P} - \vec{A}_n)^2 + \epsilon_N \tag{9}$$

where we denote the diagonal component of the Berry phase (or more precisely Berry potential) A_{nn} by A_n. If the electronic eigenvalues are degenerate, *i.e.*, to the particular eigenvalue ϵ_n correspond G_n eigenvectors, instead of one Berry phase we obtain the whole set of the $G_n \times G_n$ Berry phases, forming the matrix

$$A^{k,k'}_n = i < n,k|\nabla|n,k' > \qquad k,k' = 1,2,...G_n\,. \tag{10}$$

The gauge field so generated is in this case non-abelian and corresponds to the gauge group $U(G_n)$. In practical calculations, one truncates the infinite sum in (4) to a few finite terms. Usually the sum is taken over the degenerate subspace corresponding to the particular eigenvalue ϵ_n. This is so-called Born-Huang approximation, which we will use throughout this paper.

Let us finally note that the above formalism may be rewritten in the Lagrangian language. The corresponding effective Lagrangian is then equal to[2]

$$L^{eff}_{nm} = \frac{1}{2} M \dot{\vec{R}}(t)^2 \delta_{mn} + i \vec{A}_{mn}[\vec{R}(t)] \cdot \dot{\vec{R}}(t) - \epsilon_m \delta_{mn}. \tag{11}$$

Let us see how this scenario works for the case of the simple diatomic molecule. The fast variable describes the motion of the electron around the internuclear axis. The slow variables are the vibrations and rotations of the internuclear axis. This case corresponds to the situation when the energy of the spin-axis interaction is large compared with the energy splittings between the rotational levels. This case is usually called "Hund case a." We follow the standard textbook notation of Ref.[14]. Let $\vec{N}$ be the unit vector along the internuclear axis. We can define then the following quantum numbers

$$\begin{aligned} \Lambda &= \text{eigenvalue of } \vec{N} \cdot \vec{L} \\ \Sigma &= \text{eigenvalue of } \vec{N} \cdot \vec{S} \\ \Omega &= \text{eigenvalue of } \vec{N} \cdot \vec{J} = |\Lambda + \Sigma| \end{aligned} \tag{12}$$

so Λ, Σ, Ω are the projections of the orbital momentum, spin and total angular momentum of the electron on the molecular axis, respectively.

Let us analyze the simple case of $\Sigma = 0, \Lambda = \Omega = 1$. The fast eigenstates are

$$| \pm \Omega, \theta, \phi >_S = e^{-i\phi J_3} e^{-i\theta J_2} e^{+i\phi J_3} | \pm \Omega, 0, 0 > \tag{13}$$

where the index S denotes the parametrization singular on the south pole ($\theta = \pi$). Alternatively, we may use the parametrization

$$| \pm \Omega, \theta, \phi >_N = e^{-i\phi J_3} e^{-i\theta J_2} e^{-i\phi J_3} | \pm \Omega, 0, 0 > \tag{14}$$

which is singular on the north pole ($\theta = 0$). The Berry connection is, in our case, a 2×2 matrix with the following structure

$$\begin{aligned} A^{\Omega\Omega'}_S &= i_S\langle \pm\Omega', \theta, \phi | d | \pm \Omega, \theta, \phi \rangle_S \\ &= i_S\langle \pm\Omega', \theta, \phi | \frac{\partial}{\partial\theta} | \pm \Omega, \theta, \phi \rangle_S d\theta \\ &\quad + i_S\langle \pm\Omega', \theta, \phi | \frac{\partial}{\partial\phi} | \pm \Omega, \theta, \phi \rangle_S d\phi \,. \end{aligned} \tag{15}$$

[2]One can avoid the matrix-valued Lagrangian by using Grassmannian variables [12]. This formalism was used in [13] for describing the same molecular system.

We can use the orthonormal basis $\vec{A}_S = a_r\vec{e}_r + a_\theta\vec{e}_\theta + a_\phi\vec{e}_\phi$. A simple calculation shows that only the ϕ component is different from zero, and has the quasi-abelian form

$$a_\phi^{\Omega,\Omega'} = -\Omega\frac{1-\cos\theta}{\sin\theta}(\sigma_3)^{\Omega,\Omega'} \tag{16}$$

where σ_3 denotes the third Pauli matrix, and their components are numbered by $\Omega, \Omega' = \pm 1$. An identical calculation based on the parametrization (14) leads to the expression

$$a_\phi^{\Omega,\Omega'} = +\Omega\frac{1+\cos\theta}{\sin\theta}(\sigma_3)^{\Omega,\Omega'}\,. \tag{17}$$

We can now calculate the curvature of the Berry connection, i.e. $F = dA + A\wedge A$. In our simple case the field tensor is quasi-abelian. We can use any of the gauge fields to calculate the field tensor. The answer is given by

$$\vec{B} = \mathrm{rot}\vec{A} = -\Omega\frac{\vec{N}}{R^3}\sigma_3\,. \tag{18}$$

This is nothing else but the magnetic field of the Dirac monopole with the charge $eg = -\Omega$. We know that the monopole leads to the observable effects. The kinematical angular momentum operator gets modified due to the angular momentum stored in the field of the monopole. A short calculation allows us to extract from the canonical form (9) the rotational part of the spectrum. The effective Hamiltonian reads

$$H^{eff} = \frac{1}{2MR^2}(\vec{J}^2 - \Omega^2) + \cdots \tag{19}$$

where angular momentum operator is given by

$$\vec{J} = \vec{R}\times\vec{\Pi} - \frac{1}{2}\vec{R}\epsilon_{abc}R_aF_{bc} = \vec{R}\times\vec{\Pi} - \Omega\vec{N} \tag{20}$$

and the ellipsis denotes the vibrational terms.

Of course, the traditional calculation (as one sees in *e.g.* [14] after correcting a misprint of the factor of 2 in eq. (83.7)) leads to the identical result, modulo some phenomenological assumptions about the possible spin structure. We presented the above calculation for two reasons. Firstly, we believe that this example explains the basic features of the Berry phase, and while providing a new insight into the structure of the spectrum of the diatomic molecules, allows us to understand the modification of the rotator spectrum in terms of simple physical properties of the Dirac monopole. Secondly, in the following chapters, we will basically exploit the same strategy to construct a tower of excited states in the bag model and to make a model-independent analysis of the soliton-heavy meson

bound systems. Since the generic structure of the obtained spectra for elementary particles is basically similar to the diatomic mass formulae – modulo some generalizations due to truly non-abelian character of the phase which we will shortly sketch– we will use the above example as a guide in developing the framework for describing the technically more complicated systems one encounters in strong interaction physics.

Before leaving this section, we briefly discuss how the above discussion can be generalized to a nonabelian situation. The abelian monopole spectrum corresponds to a special case of diatomic molecule when one restricts the consideration to the degenerate Π doublet with $\pm\Omega$, $\Omega = 1$. For small internuclear distance R, the potential energy curve for the singlet Σ with $\Omega = 0$ lies higher than that for the Π for which the quasi-abelian approximation is reliable. However if R is sufficiently large, then the two potential energy curves can substantially overlap in which case one must treat the triplets (Π, Σ) together, as pointed out by Zygelman [10]. The resulting Berry potential is then truly nonabelian. The resulting spectrum can then be written in a *generic* form as [13]

$$H^{eff} = \frac{1}{2MR^2}\left(\vec{J}_{\rm R} + (1-\kappa)\vec{J}_g\right)^2 - \frac{1}{2MR^2}(1-\kappa)^2 \tag{21}$$

where $\vec{J}_{\rm R}$ is the rotor ("dumb-bell") angular momentum $\vec{R}\times\vec{\Pi}$ and $\vec{J}_g$ the angular momentum stored in the nonabelian gauge field, none of which is conserved separately and the constant κ defined by

$$\kappa = <\Pi|\frac{1}{\sqrt{2}}(L_x - iL_y)|\Sigma> \tag{22}$$

where $\vec{L}$ is the electronic orbital angular momentum, measures how much the rotational symmetry is restored, *e.g.*, $\kappa = 1$ corresponding to the full restoration of the symmetry. The conserved angular momentum is $\vec{J} = \vec{J}_{\rm R} + \vec{J}_g$ which as shown first by Jackiw [11] is independent of the charge $[1-\kappa]$. The limit $\kappa \to 0$ (small R) corresponds to the quasi-abelian magnetic monopole spectrum (19) with $\Omega = 1$. In the limit $R \to \infty$, the singlet Σ becomes degenerate with the doublet Π and hence $\kappa \to 1$. Zygelman shows that in that limit

$$1 - \kappa \sim C/R^4 \tag{23}$$

where C a constant. In this limit, one can show that the field strength tensor vanishes (pure gauge). Nonetheless as noted above, there is an angular momentum associated with the electronic degrees of freedom which however decouples from the spectrum. What happens is that the electronic rotational symmetry, broken for small R, is restored for large R so that the electronic angular momentum becomes a good quantum number. This point will be relevant when discussing the analogy with the heavy quark limit below.

3 Berry Phase in the Topological Chiral Bag

3.1 Toy Model

To fully appreciate the generic structure of the Berry potentials that we will exhibit, it is useful to reformulate a well-studied case in a way suitable to our strong-interaction model. Consider a system of slowly rotating solenoid coupled to a fast spinning object (call it "electron") described by the (Euclidean) action [15]

$$S_E = \int dt \left(\frac{\mathcal{I}}{2} \dot{\vec{n}}^2 + \psi^\dagger (\partial_t - \mu \hat{n} \cdot \vec{\sigma}) \psi \right) \tag{24}$$

where $n^a(t)$, a=1,2,3, is the rotator with $\vec{n}^2 = 1$, $\mathcal{I}$ its moment of inertia, ψ the spinning object ("electron") and μ a constant. We will assume that μ is large so that we can make an adiabatic approximation in treating the slow-fast degrees of freedom. We wish to calculate the partition function

$$Z = \int [d\vec{n}][d\psi][d\psi^\dagger] \delta(\vec{n}^2 - 1) e^{-S_E} \tag{25}$$

by integrating out the fast degree of freedom ψ and $\psi^\dagger$. This system in the space of the rotating solenoid gives precisely the same abelian monopole spectrum (19) with $\Omega = 1/2$. We will solve this problem first in the standard way used by Stone and then by the method we shall use. The procedure used by Stone goes as follows. Imagine that $\vec{n}(t)$ rotates slowly. At each instant $t = \tau$, we have an instantaneous Hamiltonian $H(\tau)$ which in our case is just $-\mu \vec{\sigma} \cdot \hat{n}(\tau)$ and the "snap-shot" electron state $|\psi_0(\tau)\rangle$ satisfying

$$H(\tau)|\psi^0(\tau)\rangle = \epsilon(\tau)|\psi^0(\tau)\rangle. \tag{26}$$

In terms of these "snap-shot" wave functions, the solution of the time-dependent Schrödinger equation

$$i\partial_t |\psi(t)\rangle = H(t)|\psi(t)\rangle \tag{27}$$

is

$$|\psi(t)\rangle = e^{i\gamma(t) - i\int_0^t \epsilon(t')dt'} |\psi^0(t)\rangle. \tag{28}$$

Note that this has, in addition to the usual dynamical phase involving the energy $\epsilon(t)$, a nontrivial phase $\gamma(t)$ – known as Berry phase – which substituted into (27) is seen to satisfy

$$i\frac{d\gamma}{dt} + \langle \psi^0 | \frac{d}{dt} \psi^0 \rangle = 0. \tag{29}$$

This allows us to do the fermion path integrals to the leading order in adiabaticity and to obtain (dropping the trivial dynamical phase involving ϵ)

$$Z = const \int [d\vec{n}]\delta(\vec{n}^2 - 1)e^{-S^{eff}}, \tag{30}$$

$$S^{eff}(\vec{n}) = \int \mathrm{L}^{eff} = \int [\frac{\mathcal{I}}{2}\dot{\vec{n}}^2 - i\vec{\mathcal{A}}(\vec{n}) \cdot \dot{\vec{n}}]dt \tag{31}$$

where

$$i\vec{\mathcal{A}}(\vec{n}) = -\langle\psi^0(\vec{n})|\frac{\partial}{\partial\vec{n}}\psi^0(\vec{n})\rangle \tag{32}$$

in terms of which γ is

$$\gamma = \int \vec{\mathcal{A}} \cdot d\vec{n}. \tag{33}$$

$\mathcal{A}$ so defined is the Berry potential or connection and γ is the Berry phase. $\mathcal{A}$ is a gauge field with coordinates defined by $\vec{n}$.

We can obtain the same result by defining $S(\tau)$ in (24) as

$$\hat{n}(\tau) \cdot \vec{\sigma} = S(\tau)\sigma_z S^\dagger(\tau). \tag{34}$$

We now rotate the electron field as

$$\psi \to S\psi. \tag{35}$$

Then Eq. (24) can be written

$$S_E = S_{S=1} + \int dt \ \left(\psi^\dagger S^\dagger i\partial_t S\psi\right). \tag{36}$$

When the electron field is integrated out, the second term of this action gives rise to the Berry potential term of (31) given in terms of the matrix element taken with the basis that diagonalizes the fermion term in $S_{S=1}$. If we call this basis $|\sigma_z>$ (*i.e.*, eigenstate of σ_z), then

$$i\vec{\mathcal{A}}(\vec{n}) = -\langle\sigma_z|S^\dagger\frac{\partial}{\partial\vec{n}}S|\sigma_z\rangle. \tag{37}$$

This is the procedure that we will use for the more complicated case of the topological chiral bag.

3.2 Bag Model

The simple description outlined above carries through in spirit to a system of quarks confined inside a cavity wrapped by a strong pion field. In the bag, the monopole field is substituted by an induced instanton-like field in isospin space (the slow variable space) and the heavy fermion is played by valence quarks. In what follows, we will use the action formulation.

Inside the bag, the quarks are described by free QCD and confined by fiat at the bag surface. To prevent explicit chiral symmetry breaking a pion field surrounds the bag. In the topological bag model the pion field has the structure of the Skyrme hedgehog ansatz. The latter is invariant under a grand-spin rotation (angular momentum plus isospin), $\vec{K} = \vec{J} + \vec{I}$. As a result, the quarks inside the cavity are polarized in a level structure that depends explicitly on the strength of the pion field at the bag surface (denoted F and referred to as "chiral angle"). The level degeneracy is $2K + 1$. Thorough discussions on the topological bag model can be found in Ref. [5, 16].

Suppose that we adiabatically rotate the bag in space. Because of the degeneracy of the Dirac spectrum, mixing between quark levels is expected no matter how small the rotation is. This mixing takes place in each quark K-band and leads to a nonabelian Berry or gauge field. Indeed, an adiabatically rotating bag can be described by the following action

$$S_S = \int_V \overline{\psi} i\gamma^\mu \partial_\mu \psi - \frac{1}{2} \int \Delta_s \, \overline{\psi} \, S \, e^{i\gamma_5 \vec{\tau}\cdot\hat{r}F(r)} \, S^\dagger \, \psi + S_M(SU_0S^\dagger) \tag{38}$$

where V is the bag volume (which we shall suppress below unless ambiguity arises), F is the chiral angle appearing in $U_0 = e^{i\vec{\tau}\cdot\hat{r}F(r)}$, Δ_s is a surface delta function and the space rotation has been traded in for an isospin rotation ($S(t)$) due to the hedgehog symmetry considered here. The purely mesonic terms outside the bag are described by S_M. Presently we shall discuss the effect of rotations on quarks only, relegating the discussion of the mesonic cloud to the second part of this chapter. The massless quarks inside the bag are assumed to be free. The rotation at the boundary can be unwound by the redefinition $\psi \to S\psi$, leading to

$$S_S = S_{S=\mathbf{1}} + \int \psi^\dagger \, S^\dagger i\partial_t S \, \psi \, . \tag{39}$$

The effect of the rotation on the fermions inside the bag is the same as a time dependent gauge potential. This is the origin of the induced Berry potential analogous to the solenoid-electron system , Eq.(36).

To understand the physics behind this term, we expand the fermionic fields in the complete set of states ψ_{KM} with energies ϵ_K in the *unrotating bag* corresponding to the action $S_{S=1}$ in (39), and M labels $2K+1$ projections of the grand spin K. Generically,

$$\psi(t,x) = \sum_{K,M} c_{KM}(t)\psi_{KM}(x) \tag{40}$$

where the c's are Grassmannians, so that

$$S_S = \sum_{KM} \int dt\ c^\dagger_{KM}(i\partial_t - \epsilon_K)c_{KM} + \sum_{KMK'N} \int dt\ c^\dagger_{KM} A^{KK'}_{MN}\, c_{K'N} \tag{41}$$

where

$$A^{KK'}_{MN} = \int_V d^3x\ \psi^\dagger_{KM} S^\dagger i\partial_t S \psi_{K'M}\,. \tag{42}$$

No approximation has been made up to this point. If the A of (42) were defined in the whole K space, then A takes the form of a pure gauge and the field strength tensor would be identically zero [3]. However we are forced to truncate the space. As in the preceding chapter, we can use now the adiabatic approximation and neglect the off-diagonal terms in K, *i.e.*, ignore the effect of adiabatic rotations, which can cause the jumps between the energy levels of the fast quarks. Still, for every $K \neq 0$ the adiabatic rotation mixes $2K+1$ degenerate levels corresponding to the particular fast eigenenergy ϵ_K. In this form we clearly see that the rotation induces a hierarchy of Berry potentials in each K-band, on the generic form identical to Eq.(10). This field is truly a gauge field. Indeed, any local rotation of the $\psi_{KM} \to D^K_{MN}\psi_{KN}$ where D^K is a $2K+1$ dimensional matrix spanning the representation of rotation in the K-space, can be compensated by a gauge transformation of the Berry potential

$$A^K \to D^K(\partial_t + A^K)D^{K\dagger} \tag{43}$$

leaving S_S invariant [17].

The structure of the Berry potential depends on the choice of the parametrization of the isorotation S (gauge freedom). For the parametrization $S = a_4 + i\vec{a}\cdot\vec{\tau}$ with the unitary

[3] As we saw in the case of the diatomic molecule, the vanishing of the field tensor does not imply that there is no effect. It describes the restoration of certain symmetry. See later a similar phenomenon in heavy-quark baryons.

constraint $a \cdot a = 1$ (unitary gauge), we have

$$A^K = T^a_K \, A^a_K = T^a_K \left(g_K \, \frac{\eta^a_{\mu\nu} \, a_\mu da_\nu}{1 + a^2} \right) \tag{44}$$

where η is the t'Hooft symbol and g_K the induced coupling to be specified below. The T's refer to the K-representation of SU(2), the group of isorotations. In the unitary gauge the Berry potential has the algebraic structure of a unit size instanton in isospace, *i.e.*, the space of the slow variables. It is not the Yang-Mills instanton, however, since the above configuration is not self-dual due to the unitarity gauge constraint. This configuration is a non-abelian generalization of the monopole-like solution present in the diatomic molecular case.

To make our analogy more quantitative, let us refer to the Grassmannians c in the valence states by α's and those in the Dirac sea by β's. Clearly (41) can be trivially rewritten in the form

$$\begin{aligned} S_S &= \sum_{KMN} \int dt \; \alpha^\dagger_{KM} \left[(i\partial_t - \epsilon_K) \mathbf{1}_{MN} - (A^K)_{MN} \right] \alpha_{KN} \\ &+ \sum_{KMN} \int dt \; \beta^\dagger_{KM} \left[(i\partial_t - \epsilon_K) \mathbf{1}_{MN} - (A^K)_{MN} \right] \beta_{KN} \, . \end{aligned} \tag{45}$$

Integrating over the Dirac sea *in the presence of valence quarks* yields the effective action

$$\begin{aligned} S_S = & \sum_{KMN} \int dt \; \alpha^\dagger_{KM} \left[(i\partial_t - \epsilon_K) \mathbf{1}_{MN} - (A^K)_{MN} \right] \alpha_{KN} \\ + & \; i \mathrm{Tr} \ln \left((i\partial_t - \epsilon_K) \mathbf{1}_{MN} - (A^K)_{MN} \right) \end{aligned} \tag{46}$$

where the Trace is over the Dirac sea states. The latter can be Taylor expanded in the isospin velocities $\dot{a}_\mu$ in the adiabatic limit,

$$i \mathrm{Tr} \ln \left((i\partial_t - \epsilon_k) \mathbf{1}_{MN} - (\mathcal{A}^K_\mu)_{MN} \dot{a}_\mu \right) = \int dt \, \frac{\mathcal{I}_q}{2} \dot{a}_\mu \dot{a}_\mu + \cdots \tag{47}$$

We have exposed the velocity dependence by rewriting the form $A^K_{MN} = (\mathcal{A}^K_\mu)_{MN} \dot{a}_\mu$. Linear terms in the velocity are absent since the Berry phases in the sea cancel pairwise in the SU(2) isospin case under consideration. For SU(3) they do not and are at the origin of the Wess-Zumino term. The ellipsis in (47) refers to higher derivative terms. $\mathcal{I}_q$ is the moment of inertia of the bag. We do not need the explicit form of this term for our considerations. We would like to point out that this term includes implicitly the valence quark effect, because the levels of the Dirac sea are modified due to the presence of the valence quarks.

To see the general motivation for studying the excited states via Berry phases let us consider the case of the bag containing one valence quark in the $K = 1$ state. The action for the adiabatic motion of this quark is obtained from the above formulae and yields

$$S_S = \int dt \; [i\alpha^\dagger_{1M}\dot{\alpha}_{1M} - \epsilon_1\alpha^\dagger_{1M}\alpha_{1M} + \frac{1}{2}\mathcal{I}_q\dot{a}_\mu\dot{a}_\mu + \dot{a}_\mu(\mathcal{A}^1_\mu)_{MN}\alpha^\dagger_{1M}\alpha_{1N}] \, . \tag{48}$$

As we will see below, when canonically quantized, the generic structure of the resulting Hamiltonian is identical to (19) and shows that the excited quark system in the slow variable space behaves as a spinning charged particle coupled to an instanton-like gauge field centered in an S^3 sphere in the four dimensional isospin space. This once again illustrates the universal character of the Berry phases.

Let us now quantize the system. Since S^3 is isomorphic to the group manifold of $SU(2)$, it is convenient to use the left or right Maurer-Cartan forms as a basis for the vielbeins (one-form notation understood)

$$S^\dagger i dS = -\omega_a\tau_a = -v^c_a(\theta)d\theta^c\tau^a \tag{49}$$

where we expressed the "velocity" forms ω in the basis of the vielbeins v^c_a, and θ denotes some arbitrary parametrization of the $SU(2)$, *e.g.* Euler angles. In terms of the vielbeins, the induced gauge potential simplifies to

$$\mathcal{A}^c = -\, g_K \, v^c_a(\theta)T^a \tag{50}$$

where T are the generators of the Berry potential in the K representation and g_K is the corresponding charge [4]

$$g_K = \frac{1}{K}\left(\frac{1}{1+y}\right) - \frac{1}{K+1}\left(\frac{y}{1+y}\right) \tag{51}$$

where

$$y = \frac{j^2_{K+1} + j^2_K - 2(K+1)j_{K+1}j_K/x}{j^2_{K-1} + j^2_K - 2Kj_{K-1}j_K/x} \cdot \frac{j_K(1+\frac{\sin F}{2K+1}) - j_{K-1}\cos F}{j_K(1-\frac{\sin F}{2K+1}) + j_{K+1}\cos F}$$

and j_K are the spherical Bessel functions calculated at $x = \omega R$ – the lowest energy solution for fixed K and parity $P = (-1)^{K+1}$ in a spherical bag. A qualitative behavior of the Dirac spectrum and the induced charge versus F are shown in Figs. 1 and 2. (Note that we could have equally well used the right-invariant Maurer-Cartan form instead of the left-invariant Maurer-Cartan form (49)). The field strength can be written in terms of $\mathcal{A}$ defined in eq.(50)

$$\mathcal{F}_K = d\mathcal{A}_K - i\mathcal{A}_K \wedge \mathcal{A}_K = -g_K(1 - g_K/2)\epsilon^{mij}T^m_K \, v^i \wedge v^j. \tag{52}$$

$\mathcal{F}_K$ vanishes for $g_K = 0$ (trivial case) and for $g_K = 2$, *i.e.* the Berry potential becomes a pure gauge.

The vielbeins – and hence $\mathcal{A}$ and $\mathcal{F}$ – are frame-dependent, but to quantize the system, no specific choice of framing is needed. The canonical momenta are $p_a = \partial L/\partial\dot{\theta}_a$. Our system lives on S^3 and is invariant under $SO(4) \sim SU(2) \times SU(2)$. Right and left generators are defined as

$$\begin{aligned} R_a &= u_a^c\, p_c \\ L_a &= D_{ab}(S)\, R_b \end{aligned} \tag{53}$$

where $u_i^a\, v_c^i = \delta_c^a$ and $D(S)$ spans the adjoint representation of the $SU(2)$. Following the procedure described in [4], we get our Hamiltonian in terms of the generators[4]

$$H^* = \epsilon_K\, \mathbf{1} + \frac{1}{8\mathcal{I}}\, (R_j - g_K\, T_{Kj})\, (R_j - g_K\, T_{Kj})\,. \tag{54}$$

This resembles closely the nonabelian molecular Hamiltonian (21). In fact, it is identical to it with a suitable reinterpretation of the charge g_K to be explained below. As a result, the Hamiltonian for a singly excited quark[5] takes the simple form

$$H^* = \epsilon_K \mathbf{1} + \frac{1}{8\mathcal{I}} \left(\vec{R}^2 - 2 g_K \vec{R} \cdot \vec{T}_K + g_K^2 \vec{T}_K^2 \right). \tag{55}$$

The spectrum can be readily constructed if we notice that (55) can be rewritten solely in terms of the independent Casimirs

$$H^* = \epsilon_K \mathbf{1} + \frac{1}{2\mathcal{I}} \left[+ \frac{g_K}{2} \vec{J_K}^{\,2} + (1 - \frac{g_K}{2}) \vec{I}^2 - \frac{g_K}{2} (1 - \frac{g_K}{2}) \vec{T_K}^{\,2} \right] \tag{56}$$

where $\vec{J}_K = -\vec{R}/2 + \vec{T}_K$ and $\vec{I} = \vec{L}/2$ are the angular momentum and isospin respectively.

The identification of the quantum numbers follows from the original symmetries of the action. Indeed, under an isospin transformation

$$S \to e^{-iT\cdot\alpha} S \qquad\qquad \psi \to \psi \tag{57}$$

[4] Canonical quantization for this system goes much like that of eq.(11) except that here we carry along Grassmanians which play an inert role of specifying the quark states involved, i.e., equivalent to projection operators. On the other hand, one can also get eq.(54) following the quantization procedure described in [12] for a system with Grassmanian variables.

[5] If we were to add a second quark to this band (doubly excited state) then we could no longer have an irreducible representation of T_K but a reducible representation instead.

following the redefinition $\psi \to S^\dagger \psi$ (isospin co-moving frame). The isospin operator is given by the standard Noether construction

$$I^a = D^{ab}(S)\left(\mathcal{I}\omega^b + \int d^3x \psi^\dagger T^b \psi\right) = D^{ab}(S)\left(\mathcal{I}\omega^b + g_K \frac{T_K^a}{2}\right). \tag{58}$$

The second term in (58) is the induced Berry phase. The term in bracket is the momentum canonically conjugate to the velocity ω^a, referred to as p^a above in the canonical frame. Under a rotation,

$$S \to S\, e^{-iT\cdot\beta} \qquad\qquad \psi \to \left(e^{iT\cdot\beta} e^{-iJ\cdot\beta}\right)\psi. \tag{59}$$

Again, the angular momentum is given by the conventional Noether construction

$$J^a = -\mathcal{I}\omega^a + \int d^3x \psi^\dagger \left(L^a + \frac{\sigma^a}{2}\right)\psi = -\left(\mathcal{I}\omega^a + g_K \frac{T_K^a}{2}\right) + \int d^3x \psi^\dagger K^a \psi. \tag{60}$$

Since the states are eigenstates of K, the last term in (60) is just the representation of the SU(2) algebra spanned by K,

$$J^a = -\left(\mathcal{I}\omega^a + g_K \frac{T_K^a}{2}\right) + T_K^a. \tag{61}$$

The angular momentum gets an extra contribution due to the induced non-abelian Berry phase. This is the reason why we are able to avoid the Skyrme constraint $I = J$ - isospin hidden in the K structure of the rotated degenerate levels gets transmuted into an extra component of the angular momentum.

For $g_K = 0$, we have the rotor spectrum $H^* = \vec{I}^2/2\mathcal{I}$. This happens for any value of the chiral angle only for the $K = 0$ level and corresponds to the known case of the nucleon and delta. For $K > 0$, g_K vanishes for some specific values of the chiral angle (see Fig.2), most probably connected with the additional level crossings in the spectrum (see Fig.1). But these may be artifacts and may not be physically meaningful. For $g_K = 2$, the Berry field strength vanishes but the gauge field nontrivially affects the spectrum, *i.e.*, $H^* \sim \vec{J}_K^2/2\mathcal{I}$. The spectrum may look analogous to the quasi-abelian case of the diatomic molecule but because of the vanishing field strength, the analogy is not significant. In our system, however, this situation is never reached as the charge g_K in Eq. (51) cannot reach 2. The reason is that there is no limit in which the adiabatic rotation would be induced by

the K-spin, and not by the isospin only. Indeed, if that was the case, we see immediately from (61) that the angular momentum of the system would reduce to the inertial part $\mathcal{I}\vec{\omega}$ carried solely by the hedgehog core. More discussions on this difference will be given in Appendix A.

There is an amusing analogy with the diatomic molecule above and the heavy quark system below, if we were to consider the fictitious situation of two quarks in the $(1^-, 2^-)$, $(2^+, 3^+)$, *etc.* states. These multiplets, correspond respectively to a core with angular momentum $\frac{3}{2}^-$, $\frac{5}{2}^+$, *etc.* coupled to isospin 1/2. They are the equivalent of the heavy quark multiplets to be discussed below. As the bag radius is increased (MIT limit) angular momentum becomes a good quantum number. Thus the isospin triplet and singlet states become degenerate. In this limit, the Berry phase stemming from the singlet *exactly* balances the Berry phase from the triplet at the MIT point ($F = 0$) since $g_2^- = -g_1^- = 1/2$, $g_3^+ = -g_2^+ = 1/3$, *etc.*. This cancellation does not occur in the lowest multiplet $(0^+, 1^+)$ with an angular momentum core $\frac{1}{2}^+$. The reason is that the Berry phase vanishes identically in the $K = 0$ state for all values of the pion field F. Since they interpolate between positive and negative energy levels, these states are not allowed to carry a Berry phase.

To summarize: We see that the role of the induced gauge potential is to lift the degeneracy between angular momentum and isospin, and leads naturally to the description of excited states. The Hamiltonian (56) allows a simple description of the even/odd parity excitations of the nucleon and Δ, in terms of the original splittings in the topological bag model and the induced Berry charge g_K. For that we have to add two quarks in the inactive band $K = 0$ each with energy ϵ_0 and recall that the parity assignment follows from the parity of the excited quark in the active band $K = 1$, which is assumed to describe the low-lying excited states.

3.3 Light-quark spectrum

A number of relations among the low-lying excited states of baryons follow from (56). Here we will only quote some model-independent results [6], obtained by elimination of both the Berry charge g_1 and the moment of inertia $\mathcal{I}$. For instance, in the Roper channel, it follows from (56) that

$$M(P11) - M(N) = M(P33) - M(\Delta). \tag{62}$$

Empirically, the left-hand side is 502 MeV and the right-hand side is 688 MeV. In the

[6] In deriving these formulae we have assumed that the pion cloud outside the bag is not substantially distorted by the excitation of a single quark inside the bag.

odd-parity channel

$$M(D13) - M(D35) + M(\Delta) - M(N) = -\frac{1}{4}(M(D35) - M(S31)). \tag{63}$$

From the data, we get 116 MeV for the left-hand side and 76 MeV for the right-hand side. Also

$$M(S31) - M(S11) = \frac{5}{2}(M(\Delta) - M(N)) - \frac{3}{2}(M(D35) - M(D13)). \tag{64}$$

Empirically, the left-hand side gives 85 MeV and the right-hand side gives 125 MeV.

We recall that the above formulae were obtained for the quark sector only (the interior of the bag), *i.e.*, till now we were ignoring the pionic cloud *outside* the bag, described by S_M in (38). We expect that the detailed analysis of the pionic sector should give the same structure of the mass formula. The argument is as follows. Description of the resonances in the Skyrme model is obtained by studying phase shifts of the pionic fluctuations in the background of the static soliton. The adiabatic rotation (cranking) of the soliton corresponds to slow variables. The pionic fluctuations are fast and are equivalent to the "particle-hole" vibrations in the quark bag. Again, the generic Born-Oppenheimer scenario tells us that the evolution of the Skyrme cloud outside the bag will be influenced by the presence of the magnetic force coming from the integrated-out pionic fluctuations. The counterpart of the charge g_K and moment of inertia will of course depend on the version of the Skyrme Lagrangian used, but the generic formula should be identical. If we neglect the anharmonicities coming from the vibration and higher order terms ($O(1/N_C^2)$) coming from the collective rotations we are at the same level of accuracy in the $1/N_c$ expansion on both sides of the bag wall, *i.e.*, in the quark sector inside as well as in the pionic sector outside the bag. The analysis done recently in [18] for the pure skyrmion case supports this point of view.

Pure skyrmion may be viewed as the limiting case of the shrinking bag. The formulae presented in [18] for the S-wave pion-nucleon scattering have the same generic form as our mass formula. We would like to stress that the proper inclusion of the rotational effects is crucial for the solution of the long standing problem in the Skyrme like models of the $S11$ and $S31$ degeneracy. Explicit calculations in [18] (although not relating explicitly to Berry phases) and our formula (64) confirm the role of the Berry phase for splitting the degeneracy between these two levels. It was noted recently by Masak *et al* [19] that incorporation of the vector mesons ρ and ω in a way consistent with hidden gauge symmetry of chiral Lagrangians [20], in particular in the intrinsic-parity odd sector, improves markedly the phase shifts for $S11$ and $S31$. The corresponding processes inside the bag would require

additional structure than what we have been considering and will bring modification to the spectrum, particularly to (64).

Finally, let us speculate how bag-radius independent the above formulae are. In other words, does the Cheshire Cat Principle [21] ("physics is independent of the bag radius") holds for the excited states? The structure of the energy levels in the bag as a function of the skyrmion profile is very complicated. When changing the bag radius, several level-crossings are expected to generate additional contributions to the induced potential. It can be shown (see Appendix A) using the reasoning of [8] that modulo a phase the same field strength tensor can be obtained either from the Berry potentials constructed within one K-subspace – as in our case – or from the off-diagonal potentials, connecting different K-subspaces. Therefore, in principle, for a large bag the spacing between the energy levels becomes increasingly small, so that some off-diagonal contributions from *e.g.* the $K = 0^+, 2^+$ levels crossing could play an important role. The point we wish to make is that the universal character of the Berry phases leaves some hope that if all the contributions to the gauge potentials are taken into account on both sides of the bag to the same order of N_c expansion, one might expect to obtain an approximate Cheshire Cat picture for the excited states at the level of the accuracy of the $1/N_c$ expansion.

4 Berry Phase in Strange Solitons

Another interesting application of the above concept is to a system composed of a soliton and a strange meson. Strange quarks play a very distinctive role in the strong interaction, being neither heavy nor light compared with the typical scale of QCD. A simple but subtle example of the interplay of strange-light degrees of freedom is provided by the Callan-Klebanov description [6] of strange baryons. In this version of the Skyrme model one assumes *ab initio* that $SU(3)$ flavor symmetry is so badly broken by the massive kaons, that the usual perturbation theory applied to the mass term in the Hamiltonian is no longer justified. Kaons are therefore described as the chiral excitations in the background of the non-strange, $SU(2)$ topological soliton. The hyperons are then described as molecule-like states composed of the kaon bound to the soliton. The identification of the quantum numbers is provided by the usual collective rotations of the soliton. Adiabatic rotation of the soliton corresponds to the slow variables, and the kaonic excitations correspond to the fast ones. We therefore could expect a Berry phase, which may influence the dynamics in a non-trivial way.

Here we will describe a simplified model for a system composed of a heavy meson

coupled to a soliton, with an overall isospin invariance. In the adiabatic limit, the system may be schematically described by

$$S_A = \int dt \left(-M_H - \frac{\mathcal{I}}{4}\mathrm{Tr}(S^\dagger \dot{S})^2 + \int d^3x K^\dagger(t,\vec{x}) \left[i\partial_t + \frac{\nabla^2}{2M_K} - SV(\vec{x})S^\dagger \right] K(t,\vec{x}) \right) \quad (65)$$

where $\mathcal{I}$ is the moment of inertia of the meson-soliton bound state, M_K is the meson mass and V is the soliton induced potential, all of which are model-dependent. Their detailed structure will not be necessary for our discussion. We will only mention that the potential distinguishes between the kaons and anti-kaons in the solitonic background. This is due to the Wess-Zumino term, which acts as a magnetic like force attracting kaons to the soliton and repulsing anti-kaons, providing in this way a mechanism for eliminating spurious states with $B = 1$, $S = 1$ from the spectrum. The Wess-Zumino term itself can be traced back as an abelian Berry phase coming from the Dirac sea of the fermionic description of the original system, but here we would like to concentrate on the Berry phase coming from the "heavy" collective quark-antiquark state (*i.e.* meson) as described above.

Again, the rotating meson background in (65) can be unwound through $K \to S(t)K$ inducing a Berry type term

$$\int dt K^\dagger (S^\dagger i\partial_t S) K \,.$$

Using the decomposition

$$K(t,\vec{x}) = \sum_n a_n(t) K_n(\vec{x}) \quad (66)$$

in the *unrotated basis*, we can rewrite (65) in the form

$$S_A = \int dt \left(-M_H - \frac{\mathcal{I}}{4}\mathrm{Tr}(S^\dagger \dot{S})^2 + \sum_{mn} {a_m}^\dagger \left[(i\partial_t - \epsilon_m)\mathbf{1}_{mn} + \int dx\, {K_m}^\dagger (S^\dagger i\partial_t S) K_n \right] a_n \right) . \quad (67)$$

The latter form is totally identical to (46) with (47) except that the a's now are c-numbers rather than Grassmannians. The role of the Berry potential is to induce hyperfine splitting in the rotor spectrum. If we denote the eigenenergy of the kaon (or more generally, the heavy pseudoscalar meson $P = K, D$ as we will discuss later) as ϵ, then the skyrmion with a bound heavy P has the fine-structure and hyperfine-structure splitting given by the Hamiltonian

$$H = \epsilon + \frac{1}{2\mathcal{I}} \left(\vec{J}_{\mathrm{R}} + c\vec{T} \right)^2 = \epsilon + \frac{1}{2I} \left(\vec{J} + (c-1)\vec{T} \right)^2 \quad (68)$$

where $\vec{J}_{\mathrm{R}}$ is the angular momentum of the rotor (related to $\vec{R}/2$ of eq.(55)), $\vec{T}$ the isospin carried by the meson (or vibration) and $\vec{J} = \vec{J}_R + \vec{T}$ is the total angular momentum of the

bound state. $\mathcal{I}$ is the moment of inertia of the rotor and c is a constant analogous to the charge $(1-\kappa)$ in diatomic molecules or to the charge $g_K/2$ of the light-quark in the chiral bag. We may immediately write the model-independent formula for this Lagrangian. It is equivalent to our formulae (62-64) and reads

$$\frac{1}{3}\left(2M(\Sigma^*)+M(\Sigma)\right)-M(\Lambda)=\frac{2}{3}\left(M(\Delta)-M(N)\right)\ . \tag{69}$$

Experimentally, the left hand side is 304 MeV and the right hand side is 293 Mev. Originally, this formula was obtained by [6] without reference to Berry phases.

5 Berry Phase in Heavy Solitons

Suppose that the strange quark mass becomes so large that it can no longer be considered as a chiral quark. The question is: As the s-quark mass increases, say, beyond the chiral symmetry breaking scale, does the concept of skyrmion with its induced gauge structure still hold? This is a relevant question since it appears now that the skyrmion picture holds even when the heavy quark becomes infinitely massive [22, 23, 24]. The correct description, however, requires starting *ab initio* with a Lagrangian that satisfies both the chiral symmetry of the light quarks and the Isgur-Wise (IW) symmetry [7, 25, 26] of the heavy quarks. The heavy-quark symmetry implies that the pseudoscalar meson P which plays a key role in the Callan-Klebanov model and the corresponding vector meson P^* of the quark configuration $Q\bar{q}$ (where Q denotes heavy quark and $q=u,d$ light quark) become degenerate.

Our starting point is the effective action for heavy-light mesons in the infinite quark mass limit. If we denote by

$$H=\frac{1+\gamma^0}{2}(-\gamma_i P_i^*+i\gamma_5 P)\qquad\text{and}\qquad \overline{H}=\gamma^0 H^+\gamma^0 \tag{70}$$

the $(0^-,1^-)$ degenerate doublet in the rest frame of the heavy quark, then to leading order in the derivative expansion the effective action follows from [26, 24]

$$\mathcal{L}_H=-i\mathrm{Tr}(\partial_t H\bar{H})+\mathrm{Tr}HV^0\bar{H}-g_H\mathrm{Tr}HA^i\sigma^i\bar{H}+m_H\mathrm{Tr}H\bar{H} \tag{71}$$

Here the vector and axial currents are entirely pionic and read

$$\begin{aligned} V_\mu &= +\frac{i}{2}\left(\xi\partial_\mu\xi^\dagger+\xi^\dagger\partial_\mu\xi\right),\\ A_\mu &= +\frac{i}{2}\left(\xi\partial_\mu\xi^\dagger-\xi^\dagger\partial_\mu\xi\right). \end{aligned} \tag{72}$$

The pion field $\xi = \exp(i\vec{\tau}\cdot\vec{n}F(r)/2)$ is described by the usual Skyrme type action. Alternative formulations involving light vector mesons are also possible in which case a term of the form

$$\sim \mathrm{Tr}\bar{H}Hv_\mu B^\mu \tag{73}$$

where B^μ is the topological baryon current can be generated [24] and provide a binding mechanism as discussed in [23]. In (71) the parameter m_H is a mass of order m_Q^0. For other conventions we refer to [24].

The effective action following from (71) is invariant under local $SU(2)_V$ symmetry (h), in which V transforms as a gauge field, A transforms covariantly and $H \to Hh^\dagger$ and $\overline{H} \to h\overline{H}$. It is also invariant under heavy-quark symmetry $SU(2)_Q$ (S), $H \to SH$ and $\overline{H} \to \overline{H}S^\dagger$. This symmetry mixes the vectors (1^-) with the pseudoscalars (0^-). Under the infinitesimal transformation,

$$\delta\vec{P}^* = \vec{\alpha}P + (\vec{\alpha}\times\vec{P}^*) \qquad\qquad \delta P = -\vec{\alpha}\cdot\vec{P}^* \tag{74}$$

In the soliton sector the pion field is in the usual hedgehog configuration. In this case it is useful to organize the H field in K-partial waves. Generically

$$H(x,t) = \sum_{KM} a_{KM}(t)H_{KM}(x) \tag{75}$$

where the a's annihilate H particles with good K-spin where $\mathbf{K} = \mathbf{I} + \mathbf{J} \equiv \mathbf{K}_L + \mathbf{S}_Q$ with $\mathbf{I}$ and $\mathbf{J}$ the total isospin and angular momentum of the H-soliton system, K_L the K spin of the light antiquark in H and $\mathbf{S}_Q$ the spin of the heavy quark. In the original approach of Callan and Klebanov [6], the $K^\pi = \frac{1}{2}^+$ state in the kaon channel was found to bind to the soliton. Can this binding persist in the infinite mass limit?

To answer this question, first let us recall the essential feature of the Callan-Klebanov scheme which we have argued above is closely connected to the gauge field hierarchies induced dynamically. In the Callan-Klebanov scenario the Wess-Zumino term plays a central role in lifting the degeneracy between the strangeness $S = \pm 1$ states and assigning the correct quantum numbers to the physical states. The presence of the Wess-Zumino term causes P-wave kaons to bind to the soliton to order N_c^0. The bound state carries the good grand spin $K = I + J$ $(\frac{1}{2}^+)$ and heavy-flavor quantum number. However states with good isospin (I) and angular momentum (J) emerge after "cranking" (or rotating) the kaon-soliton bound state as a *whole*.

The origin of the Wess-Zumino term goes back to the underlying fermionic character of all hadronic excitations. In Appendix B, we argue that as the mass of Q increases, the

topological Wess-Zumino term decouples from the heavy sector and hence one of the principal (if not *the principal*) agents for the binding needed for a skyrmion with a heavy meson is lost. The reason can be easily understood. As the mass of the strange quark is increased (say, to that of the charm or bottom quark), the Wess-Zumino term truncates to the two-flavor sector which is identically zero. This is because the heavy mesons can no longer be viewed as *angle* excitations of the chiral order parameter in the QCD vacuum. The rate at which the Wess-Zumino term disappears depends on detailed dynamics. Our qualitative arguments suggest that the rate is controlled by the ratio of the induced constituent masses σ. For strange quarks this ratio is : $\sigma/\sigma_S \sim 0.47$ while for bottom quarks this ratio is : $\sigma/\sigma_B \sim 0.32$.

What is the fate of the heavy-meson-skyrmion bound state when the Wess-Zumino term vanishes? Two mechanisms providing classical binding were proposed [24, 23]. In the first approach the binding depends on the form of background pionic potential, in the second it is strengthened by the vector-meson induced term (73). If the binding persists even in the heavy quark limit then our previous discussion carries through nicely. Indeed as the mass of the heavy quark is raised, the Berry phase receives contribution from both the P and P^*. Generically

$$H = \epsilon + \frac{1}{2\mathcal{I}}\left(\vec{J}_{\rm R} + c\vec{T} + c_*\vec{T}_*\right)^2 \tag{76}$$

where $\vec{T}_*$ is the isospin contribution of P^* to the induced Berry phase. More explicitly, this formula can be rewritten as [27]

$$H = \epsilon + \frac{1}{2\mathcal{I}}\left((\vec{\mathbf{J}} - \vec{\mathbf{S}}_H) - (1-C_P)\mathrm{Tr}(P\vec{\mathbf{I}}P^+) - (1-C_P^*)\mathrm{Tr}(P_j^*\vec{\mathbf{I}}P_j^{*+})\right)^2 \tag{77}$$

where I, J are isospin and angular momentum operators, respectively, and S_H is the total spin of the H-particle. In the $K = \frac{1}{2}^+$ shell it reduces to $\vec{\sigma}/2$, *i.e.* spin 1/2 representation. This shows that the spin of the heavy quark and the light quark *fractionate* in the K-representation. The H particle bound to the skyrmion resembles a heavy fermion with spin 1/2. A similar transmutation occurs in the Callan-Klebanov construction [6]. This fermionization of the original bosonic degrees of freedom through the hedgehog structure is what makes skyrmions so remarkable. This result carries to higher K-shells.

In general C_P and C_P^* are complicated functions of the heavy quark mass. However, in the heavy quark limit $C_P = -C_P^* = 1$ and one recovers the rotor spectrum. This cancellation is guaranteed by two facts : heavy quark symmetry that implies the same strength for C_P and C_P^* and the underlying hedgehog character of the skyrmion that forces

the isospin in P^* to be antiparallel to the spin, flipping the sign of C_P compared to C_P^*. In the infinitely heavy quark limit Hamiltonian for the $K = \frac{1}{2}^+$ shell takes the form [23, 27] (to order $m_Q^0 N_c^{-1}$)

$$H^{\frac{1}{2}} = \frac{\mathbf{J}_R^2}{2\mathcal{I}} = \frac{(\vec{\mathbf{J}} - \vec{\mathbf{S}}_H)^2}{2\mathcal{I}} = \frac{\mathbf{I}^2}{2\mathcal{I}}\,. \tag{78}$$

Thus to order $m_Q^0 N_c^{-1}$ the Σ and Σ^* are degenerate. Note that the situation is totally analogous to the non-abelian molecular case for $R \to \infty$ discussed in Section 2.

The above Hamiltonian implies the following mass relation in the heavy hyperon spectrum

$$(M(\Sigma^*) - M(\Lambda)) = \frac{2}{3}\,(M(\Delta) - M(N)) \tag{79}$$

If we were to ignore P^* for any finite m_Q then (77) reduces to

$$H_1 = \frac{1}{2\Omega}\left(\vec{\mathbf{J}} - (1 - C_P)\mathrm{Tr}(P\vec{\mathbf{I}}P^+)\right)^2. \tag{80}$$

The latter reduces to (the incorrect) $H = \mathbf{J}^2/2\Omega$ as opposed to (the correct) $H = \mathbf{I}^2/2\Omega$ in the heavy quark limit.[7]

It is interesting to ask which of the mass formulae, (69) or (79), works better for the charm sector. The direct comparison is impossible at the moment, since the mass of the Σ^* is not yet measured. The mass spectrum predicted according to the Callan-Klebanov scheme – and generalized for more than one heavy mesons – for the charm baryons is given in [29]. There the Ξ's and Ω's are described by binding the K's and D's without interactions, that is to say, in quasiparticle approximation. The prediction of [29] which does not manifestly respect the Isgur-Wise symmetry is nonetheless surprisingly close to that of quark models, suggesting that perhaps the mass of the charm quark is not large enough to see clearly the effect of the Isgur-Wise symmetry at the level of mass formulae. The *effective* hyperfine coefficient c as defined in (68) comes out to be 0.62 for the strange hyperons and 0.14 for the charmed hyperons. The latter is small, but certainly not near zero as would be the case if the charm quark were massive enough to satisfy the Isgur-Wise symmetry.

Let us finally note that an approach to the heavy solitons similar in spirit to what was discussed above was suggested recently by Manohar and collaborators [22]. The difference is that we have insisted on the mechanism of binding at the classical level (in [22] binding

[7]The heavy-meson limit of the Callan-Klebanov model with the Skyrme quartic term or with vector mesons as studied in [28, 29] do not go to $H = \mathbf{J}^2/2\Omega$ since part of the P^* contribution is included in the treatment. It does not go to the correct heavy limit either.

has quantum mechanical nature) and we rely on the concept of the Berry phases. In our approach the P and P^* are defined in the (isospin) co-moving frame making their quantization simpler for the bound state problem since they do not carry good isospin (they carry good K-spin). The *dressed* P and P^* used by Manohar and collaborators are defined in the laboratory frame and their quantization is simpler for the scattering problem since they carry good isospin (as asymptotic P and P^* do). The two descriptions are related by a global isospin rotation. Since both descriptions have built in heavy quark symmetry, they yield similar physical predictions. Indeed it is not difficult to also formulate Manohar's approach in such a way that the Isgur-Wise symmetry is realized as the vanishing of the Berry potential defined in the laboratory frame [30].

6 Conclusions

The topological bag model offers a suitable setting for discussing Berry phases. The analogy with the fermion-monopole system is striking. In the bag, the strong pion field distorts the Dirac spectrum causing the emergence of Berry phases under any adiabatic rotation. While the Dirac sea produces no net Berry (Wess-Zumino) contribution due to pairwise cancellations in the sea, the valence states do. The net effect is similar to a spinning charged particle coupled to an instanton-like gauge field in isospin space.

The role of the Berry phase is to induce hyperfine splitting in the rotor spectrum. This effect can be used to describe excited baryons in the light-quark nonstrange sector. The model-independent relations discussed here are in fairly good agreement with the data. Given the simplicity of the description this is striking.

We have argued that the features displayed in the context of the topological bag model are in fact generic. They can be easily extended to strange baryons as discussed by Callan and Klebanov and even to heavier systems as the ones discussed by Manohar and collaborators [22] and also by Min and collaborators [23]. This is hardly a surprise given the generic character of Berry phases.

Our work is certainly far from complete. We have not investigated systematically the relevance of the Cheshire Cat description for the excited states, nor have we explored totally the heavy light systems. Moreover, we should be also able to address exotic issues related to photoproduction mechanisms and dibaryon systems where excited quarks are naturally triggered. We hope, however, that our initiative will spur more excitement in these directions.

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Appendix A: The Nonvanishing of Nonabelian Berry Potentials in the Chiral Bag

In this Appendix, we wish to explain the difference in structure between the induced field in light-quark systems and the one in heavy-quark systems and also in diatomic molecules. We noted in the main text that while the structure of Berry potentials and their physical effects are generic, the field tensor behaved differently. To be specific, in heavy baryons and diatomic molecules, both the Berry potential and its field strength vanished in certain limit while they did not in the chiral bag modeling the light-quark baryons [8].

We first note that when the condition of adiabaticity is satisfied, there are two ways of describing Berry potentials. One is the standard way used by Berry [1] which is to define the potential within a diagonal subspace (denoted $\mathcal{A}$) and the other proposed by Aharanov et al. [8] is to define it in off-diagonal subspaces (denoted $\tilde{\mathcal{A}}$). To define these quantities precisely, we use the Hamiltonian formalism of Ref.[31]. Let the *fast variable* Hamiltonian parametrized by a_μ at a given time t be written in the form

$$H(a_\mu) = \sum_K \epsilon_K(a_\mu)\Pi_K(a_\mu) \tag{A.1}$$

where $\Pi_K(a_\mu)$ is the projection operator onto the subspace (labeled by the index K) spanned by the 'snap-shot' energy eigenstate of $\epsilon_K(a_\mu)$,

$$H(a_\mu)|K, a_\mu\rangle = \epsilon_K(a_\mu)|K, a_\mu\rangle. \tag{A.2}$$

The quark action in eq.(38) discussed in ref.[4] can clearly be quantized to take this generic form. In *adiabatic approximation* the standard form of the Berry potential that is inherited from the *fast* space can be written as

$$\mathcal{A} = \sum_K \Pi_K S^\dagger dS \Pi_K \tag{A.3}$$

where the dependence of the projection operator on the coordinates a_μ is suppressed and

$$|K, a_\mu(t)\rangle = S(t)|K, a_\mu(0)\rangle. \tag{A.4}$$

Let us now define formally the off-diagonal field, $\tilde{\mathcal{A}}$, that connects different subspaces

$$\tilde{\mathcal{A}} = \sum_{K\neq K'} \Pi_K S^\dagger dS \Pi_{K'}. \tag{A.5}$$

[8] It is possible to construct a chiral bag that includes heavy mesons for which case one should also have a vanishing Berry potential in heavy-meson limit. See [33] for a recent discussion on this.

This is the gauge potential of Aharanov et al. [8] up to a unitary transformation. Calculating the field strength with $\tilde{\mathcal{A}}$ we get

$$\mathcal{F}_{\tilde{\mathcal{A}}} = \sum_{K"} \Pi_{K"} \left(\tilde{\mathcal{A}} \wedge \tilde{\mathcal{A}} \right) \Pi_{K"}. \tag{A.6}$$

Thus as discussed in Ref.[8, 32], although we have used the field $\tilde{\mathcal{A}}$ that only mixes different spaces, the field tensor is diagonal. Now let us calculate the field tensor with the diagonal field $\mathcal{A}$ (A.3). Using the properties of the projection operator, one can readily verify that one obtains exactly the same expression as (A.6) except for a minus sign

$$\mathcal{F}_{\mathcal{A}} = -\mathcal{F}_{\tilde{\mathcal{A}}}. \tag{A.7}$$

This is of course a direct consequence of the fact that $\mathcal{F}$ originates from the diagonal of $S^{\dagger}dS$, and the latter is a pure gauge in the full Hilbert space.

The way this relation might impact on our discussion is as follows. In subsections (3.2) and (3.3) we focused on the $K = 1^{+}$ band which is the first excited K band above the ground band $K = 0^{+}$. However, the $K = 1$ band can be connected by the adiabatic rotation operator S to not only the $K = 0$ band but also to the $K = 2$ band. As the bag radius increases or equivalently the chiral angle F tends to zero, the $K = 0, 1$ bands cross each other (following the restoration of the angular momentum into the Dirac spectrum). The $K = 2$ band never crosses any of them at any point of the chiral angle since it carries different angular momentum. This should be contrasted with the molecular case or with the heavy-baryon case. In the diatomic molecule, the doubly degenerate Π states cross the singlet Σ at $R = \infty$ at which point the rotational symmetry is restored in $L = 1$. What (A.7) says is that sufficiently far away from the triple degeneracy point, one can describe the spectrum either with the diagonal field or with the off-diagonal field. An analogous situation holds for the heavy-baryon case where the singlet P "crosses" the triplet P^{*} in the IW limit. Thus what is different in the chiral bag case is that there is *no point* at which all the relevant K states, namely $K = 0, 1, 2$, become degenerate.

To make the above statements more quantitative, Let us make an *ansatz* for a Berry potential that captures the essence of the above structure. We take in K space

$$\mathbf{A}_{KK'} = \mathcal{A}\,\delta_{KK'} + \rho_{KK'}\tilde{\mathcal{A}}_{KK'} \tag{A.8}$$

where we have introduced the "suppression factor" $\rho_{KK'}$ for $K \neq K'$ for which we make the simplest possible assumption,

$$\begin{aligned} \rho_{KK'} &= 1, \quad \text{for } |\epsilon_K - \epsilon_{K'}| \ll \Delta, \\ &= 0, \quad \text{otherwise.} \end{aligned} \tag{A.9}$$

Here the Δ represents the scale of the adiabaticity of the slow-variable system. The standard Berry potential is recovered when the adiabatic change of state (*i.e.*, the complete suppression of off-diagonal transitions) is applicable, that is to say, $\rho = 0$. Now we calculate the field strength of $\mathbf{A}$ using eq.(A.8),

$$\begin{aligned} \mathcal{F}^K_{\mathbf{A}} &= \Pi_K \mathbf{A} \wedge \mathbf{A} \Pi_K \\ &= \sum_{K' \neq K} \left(1 - |\rho_{KK'}|^2\right) \tilde{\mathcal{A}}_{KK'} \wedge \tilde{\mathcal{A}}_{K'K} \end{aligned} \tag{A.10}$$

where the superscript K on the field strength means that we are focusing on a particular K space. Although it is obtained with a specific *ansatz*, we believe (A.10) to be generic. To see that it is quite general, consider the diatomic molecular case [10, 13]. As the internuclear distance R becomes large, the energies of the Π and Σ levels become degenerate and hence $\rho_{\Sigma\Pi} = 1$. Therefore

$$\mathcal{F}^{\Sigma} = 0 = \mathcal{F}^{\Pi}, \tag{A.11}$$

implying the vanishing of the induced interaction[9].

Let us now use eq.(A.10) to show that in contrast to the diatomic molecule, there is no such limit in the chiral bag for light-quark baryons for either the gauge field or the field tensor to vanish. Suppose such a limit existed in the topological bag model. Then from the above discussion, we should expect *all* the relevant energy levels connected to the reference K level by the adiabatic rotation operator S to become degenerate. But in the chiral bag model with the charges given by (51), this cannot happen. What happens is that when the chiral angle $F(R)$ goes to 0, the $K = 0, 1$ levels become degenerate. So from eq.(A.10), for $K = 1$,

$$\mathcal{F}^{K=1}_{\mathbf{A}} = \left(1 - |\rho_{10}|^2\right) \tilde{\mathcal{A}}_{10} \wedge \tilde{\mathcal{A}}_{01} + \left(1 - |\rho_{12}|^2\right) \tilde{\mathcal{A}}_{12} \wedge \tilde{\mathcal{A}}_{21}, \tag{A.12}$$

$$= \left(1 - |\rho_{12}|^2\right) \tilde{\mathcal{A}}_{12} \wedge \tilde{\mathcal{A}}_{21} \tag{A.13}$$

since $\rho_{10} = 1$ from our *ansatz* (A.9). However $\rho_{12} = 0$ since the $K = 2$ state is still split from $K = 0, 1$ states. Therefore (A.13) need not vanish. A similar observation can be made as the bag radius goes to zero, although the nature of level crossings is somewhat different.

In the derivation of the Hamiltonian for the excited states, eq.(56), the adiabatic approximation has been assumed to be valid. This has led to the bag-radius-independent ("Cheshire Cat") mass relations among the excited baryons as discussed in the text. The

[9] In fact, if the Π and Σ levels are degenerate, then the induced gauge potential is really a pure gauge which can be gauged away so that $\mathbf{A} = 0$ and $\mathcal{F}_{\mathbf{A}} = 0$.

repeated level crossings, however, may invalidate some of these approximations. Also, as the bag radius is increased the level spacing decreases suggesting also the breakdown of the adiabatic approximation. This seems to suggest that the so-called "bag-radius-independent" mass relations cannot hold and hence the Cheshire Cat Principle must be breaking down for the excited states. Nonetheless the mass formulas in subsection 3.3 worked fairly well. How do we understand this?

The answer may lie in the fact that there is *no limit* at which the field tensor (or the gauge potential) vanishes. At the bag radius at which the adiabaticity condition presumably fails to hold, the off-diagonal contributions could significantly modify the charge g_1 from the value implied by (51) in a way suggested by eq.(A.12). However since $(g_K)_R$ cannot 2, the structure of eq.(56) from which the same mass relations follow will remain unmodified.

Appendix B: The Vanishing of the Wess-Zumino Term for Heavy Quark

Consider QCD with two massless quarks and a heavy quark of mass m_Q. Generically, the effective action in the single gluon-exchange approximation to QCD can be rewritten as follows (using Euclidean conventions)

$$S[S,P] = -N_C \,\mathrm{Tr}\,\mathrm{Ln}\,(\not\partial + m + S + iP) \tag{A.1}$$

where S and P are scalar and pseudoscalar 3×3 hermitian matrices in flavor space and m is short for $m = m_Q(\mathbf{1} - \sqrt{3}\lambda_8)/3$. A comprehensive discussion of (A.1) can be found in Ref.[34]. Without loss of generality, we can use the decomposition

$$S + iP = \Sigma\; e^{i\gamma_5\phi^a T^a} \equiv \Sigma\; U_5^\dagger \tag{A.2}$$

Standard arguments show that the ϕ's could be interpreted as pseudoscalar mesons and that Σ can be related to the dynamically generated (or "constituent") quark mass in the vacuum [34]. Since the argument in (A.1) is nonhermitian operator, the effective action develops both a real (S_R) and imaginary part (S_I). The latter follows from

$$\frac{\delta S_I}{\delta\phi^a} = \frac{1}{2}\left((\not\partial + m + \Sigma\; U_5^\dagger)^{-1}\Sigma\frac{\delta U_5^\dagger}{\delta\phi^a} - \mathrm{h.c.}\right) \tag{A.3}$$

and gives rise "usually" to the Wess-Zumino term. If the mass of the heavy quark becomes large the Wess-Zumino term vanishes in the three flavor case.

Indeed, let us assume that the constituent masses are triggered by the quark condensation in the vacuum. The details by which this occurs is certainly model-dependent, however, the generic trend is not. Generically the quark condensate is given by

$$< \overline{\Psi}\Psi >= -i\int d\lambda \frac{1}{\lambda + im}\rho(\lambda) \tag{A.4}$$

where $\rho(\lambda)$ is the distribution of the eigenvalues of the Dirac operator in Euclidean space. For massless quarks it reduces to

$$< \overline{q}q >= -\pi \mathrm{sgn\ m}\ \rho(0), \tag{A.5}$$

whereas for heavy quarks $m_Q >> \kappa$ – the half width of $\rho(\lambda)$, typically of the order of Λ in QCD –, we have

$$< \overline{Q}Q >= -\frac{1}{m_Q}\int d\lambda\rho(\lambda)\,. \tag{A.6}$$

Typically, the eigenvalues have a Gaussian distribution (following the randomness prevailing in the QCD vacuum) so that

$$\rho(\lambda) \sim \rho(0) \ \exp \frac{-\lambda^2}{4\kappa^2} \tag{A.7}$$

leading to

$$\frac{<\overline{Q}Q>}{<\overline{q}q>} = \sqrt{\frac{2}{\pi}} \frac{\kappa}{m_Q} \tag{A.8}$$

This shows that the heavy quark condensate in the vacuum vanishes as $1/m_Q$. Up to this point, the arguments are general.

To be able to relate (A.8) to the "constituent" masses, $\Sigma = \mathrm{diag}(\sigma, \sigma, \sigma_Q)$, we need to resort to a model description of the vacuum. Sum rule arguments combined with the constituent quark model [35] suggest that

$$\frac{\sigma_Q}{\sigma} = \left(\frac{<\overline{Q}Q>}{<\overline{q}q>} \right)^{1/3} = \left(\sqrt{\frac{2}{\pi}} \frac{\kappa}{m_Q} \right)^{1/3} \tag{A.9}$$

which shows that the "constituent" mass vanishes as the inverse cubic root of the heavy current quark mass in the limit where $m_Q >> \kappa \sim \Lambda$. It is possible that other models lead to a somewhat different scaling, but we believe the estimate (A.9) is good enough to gain some idea how things might go. In the heavy quark limit $\sigma_Q \sim 0$.

With the above in mind, we can evaluate the Wess-Zumino term through the standard derivative expansion, using for the propagator

$$(\not\partial + m + \Sigma)^{-1} = \frac{\not\partial - \sigma}{\partial^2 - \sigma^2} \mathbf{1}_2 + \frac{\not\partial - m_Q}{\partial^2 - m_Q} \mathbf{1}_3 \tag{A.10}$$

where $\mathbf{1}_2 = \mathrm{diag}(1, 1, 0)$ and $\mathbf{1}_3 = \mathrm{diag}(0, 0, 1)$. Since $\mathbf{1}_2 \ \mathbf{1}_3 = 0$ it is straightforward to show that the heavy quark contribution drops from the Wess-Zumino term, and one is left only with the two-flavor (chiral quark) Wess-Zumino term that is known to vanish.

To summarize, we have shown that as the quark mass m_Q becomes considerably larger than $\kappa \sim \Lambda$ – the width of the eigenvalues distributions of the Dirac operator in the vacuum –, the heavy quark decouples and the Wess-Zumino term truncates to the two-flavor Wess-Zumino term which is identically zero. Our reasoning sketched above could presumably allow one to make an explicit calculation of the rate at which the Wess-Zumino term vanishes with the heavy-quark mass.

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FIGURE CAPTIONS

Figure 1

Schematic quark spectrum $\epsilon_K R$ (where the subscript K stands for the grand spin of the quark level) in the chiral bag wrapped by hedgehog pions as function of the chiral angle $F(R)$. Note that $F(0) = -\pi$. For a realistic spectrum, see Mulders in Ref.[16].

Figure 2

Schematic plot of the "Berry charge" g_K where K is the grand spin of the quark level as function of the chiral angle $F(R)$. For a more realistic plot, see Ref.[4].